FINAL REPORT

SUBGRADE ELASTIC MODULUS FOR ARIZONA PAVEMENTS

by

Robert W. Crossley, P. E.

&

George H. Beckwith, P. E.

Submitted to

Arizona Department of Transportation

Highways Division

for

Research Project - HPR 1-15 (156)

Sponsored by

Arizona Department of Transportation

in Cooperation with

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Federal Highway Administration

Sergent Hauskins, and Beckwith, Inc.

Consulting Geotechnical Engineers

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16. Abstract

Elastic modulus of subgrade is an important input parameter in the newly developed rational methods for the design of pavements. In order to provide guidance for the selection of realistic values of E_s for pavement design in Arizona, testing was performed at five field sites typical of Arizona subgrades. Testing involved plate bearing tests, refraction seismic, CBR, Dynaflect, and Road Rater. Correlations are made and guidelines are presented for incorporation of the findings into a pavement management system.

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ABSTRACT

Elastic modulus of subgrade is an important input parameter in the newly developed rational methods for the design of pavements. In order to provide guidance for the selection of realistic values of E_s for pavement design in Arizona, testing was performed at five field sites typical of Arizona subgrades. Testing involved plate bearing tests, refraction seismic, CBR, Dynaflect, and Road Rater. Correlations are made and guidelines are presented for incorporation of the findings into a pavement management system.

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LIST OF ABBREVIATIONS AND SYMBOLS

A	Coefficient in resilient modulus equation
a	Radius of a plate
AASHTO	American Association of State Highway and Transportation Officials
ADOT	Arizona Department of Transportation
ADT	Average Daily Traffic
ASTM	American Society for Testing Materials
B	Bulk Modulus
B	Footing Width
B	Exponent in resilient modulus equation
CBR	California Bearing Ratio
cm	Centimeter
CRT	Cathode Ray Tube
D	Confined Modulus
DSM	Dynamic Stiffness Modulus
d	Dynaflect deflection reading
d_1, d_2, etc.	Dynaflect reading based on numbered sensor
E	Young's Modulus, Modulus of Elasticity
E_s	Modulus of Elasticity of soil or subgrade*
E_{seis}	Modulus of Elasticity of soil under seismic loading
Ec	Modulus of Elasticity of soil determined by consolidation test
Ep	Modulus of Elasticity of soil determined by Menard Pressuremeter
E_{dynf}	Modulus of Elasticity of soil determined by Dynaflect
E_{RR}	Modulus of Elasticity of soil determined by Road Rater

*In Section III and IV of this text E_s is the modulus of subgrade calculated from a large (30") diameter plate bearing test under 30-50 psi.

LIST OF ABBREVIATIONS AND SYMBOLS (cont.)

E_{d_1}, E_{d_2}, etc.	Modulus of Elasticity of soil determined by Dynaflect using only the numbered sensor
E_1, E_2, etc.	Modulus of Elasticity of material in a layered elastic system
E^*_1, etc.	Elastic Modulus in layered system determined by computer iteration
$\lvert E^* \rvert$	Complex Modulus
EAL	Equivalent Axel Load
ECI	Elastic Compliance Index
ERF	Environmental Reduction Factor
^oF	Degrees Farenheit
FAA	Federal Aviation Administration
G	Shear Modulus
H	Elastic Constant in Ahlvin and Ulery expression
h_1, h_2, etc.	Thickness of layer
I_p	Influence factor
K_1	Coefficient in resilient modulus equation
K_2	Exponent in resilient modulus equation
k	Modulus of Subgrade Reaction, on spring constant of soil
k_h	Horizontal modulus of subgrade reaction
LL	Liquid Limit
M	Modulus of Deformation
MESL	Membrane Encapsulated Soil Layer
M_R	Modulus of Resilience
N	Penetration Number for blows per 6 inches
NA	Not Available
NDT	Non-destructive Testing
n_h	Rate of change of k_h with depth
P	Load on plate
PCA	Portland Cement Association

LIST OF ABBREVIATIONS AND SYMBOLS (cont.)

PI	Plasticity Index
p	Pressure on loaded area
psi	Pounds per square inch
R	Stabilometer Value
S	Spreadability
SDI	Subgrade Damping Index
SHB	Sergent, Hauskins, and Beckwith
SSV	Soil Support Value
SPT	Standard Penetration Number
V	Volume
V_o	Initial Volume
V_s	Velocity of Shear Wave
V_c	Velocity of Compression Wave
w/c	Moisture Content
γ_d	Dry density
Δ	Incremental change
ε	Strain
ε_t	Tangential Strain
ε_v	Vertical Strain
ρ	Mass Density
ρ	Deflection
θ	Sum of principal stresses (σ_1+2σ_3)
σ_1	Major Principal Stress
σ_3	Minor Principal Stress
σ_d	Deviator Stress
ν	Poisson's Ratio

PROJECT SUMMARY

The Arizona Department of Transportation is in the process of developing a pavement management system which will include rational pavement design and overlay procedures based on fundamental material properties. Elastic models will likely be used; hence a body of knowledge concerning the elastic behavior of typical subgrade soils in Arizona is needed.

In January, 1976, Sergent, Hauskins, and Beckwith, Inc., Consulting Geotechnical Engineers, was commissioned to undertake a study of subgrade elastic modulus as it pertains to pavement design in Arizona. The research effort was to focus particularly upon the response of partially saturated and cemented desert soils which are widespread in the state, and which have received little attention in the general engineering literature.

The study consists of five elements. Firstly, a literature review of the subjects of elastic properties of soils, pavement design, and environmental changes in subgrades was performed. Secondly, a sensitivity analysis and review was made to determine the importance, in terms of economy of design, of subgrade modulus as an input parameter. Thirdly, an extensive program of comparative field testing at five test stations, typical of Arizona subgrades, was formulated and performed. Testing involved repetitive and non-repetitive plate bearing tests, refraction seismic, in-situ CBR, Dynaflect, and Road Rater. The fourth element involved the correlation of data from the various tests - including comparison of findings from the five sites with findings presented by researchers in other parts of the country

and throughout the world. The fifth element consists of a series of conclusions and recommendations for incorporating the findings of this study into future pavement design and management systems.

The two year project is presented herein with the general conclusion that elastic modulus of subgrade for most typical Arizona soils is far from an intrinsic material property, but is a highly dependent variable. Due to the cemented, dessicated, and fissured state of most soils in this semi-arid environment, correlations of E_s with index properties are impractical. The most responsible approach to the selection of meaningful values of E_s for pavement design lies in the application of a statistically valid body of in-situ test data, combined with a thoughtful amount of engineering judgement and intuition.

SECTION I. INTRODUCTION, OVERVIEW, AND DISCUSSION OF THE USE OF ELASTIC MODULUS IN PAVEMENT DESIGN

1.1 Introduction

Continually rising costs coupled with an increasing scarcity of quality roadbuilding materials in many parts of the world in recent years have created a desire on the part of engineers for optimization in the design of concrete and asphalt pavements. As is the case today in many areas of engineering technology, empirical methods for pavement design, founded on what had been considered "successful" past designs, are giving way to mechanistic procedures utilizing fundamental material properties.

Today in virtually every political subdivision, responsible agencies have adopted or are in the process of developing more sophisticated procedures which seek to model pavement performance in a rational manner. In the United States, the Federal Highway Administration has urged each State Department of Transportation to develop its own pavement design and management systems, incorporating regional considerations of climate, subsurface environment and economics of construction. For several years, the Arizona Department of Transportation, in cooperation with the universities and private consultants, has been working toward developing such a system.

With very few exceptions, each of the several dozen new pavement design methods which are described in the technical literature utilize a layered systems approach to model pavement structures. Inasmuch as the behavior of a layered system depends on the stress-strain performance of the various layers, a model characterizing the load response of each material is a vital element in every pavement design method. The simplest and most widely used model for materials response is the linear elastic model, although more sophisticated models such as visco-elastic, elasto-plastic, and finite element models are also found.

In a typical mechanistic flexible pavement design procedure based on elastic theory, a three layer system consisting of an asphaltic concrete or bituminous bound layer, an unbound granular base layer, and an infinite subgrade is described. Response of each of the materials is a function of the two fundamental elastic parameters, modulus of elasticity (E) and Poisson's ratio (ν), which must be assigned to each layer. Other important input parameters include the traffic data (the number of wheel loads, size, and tire pressure), and thicknesses of the various layers. With these, the performance of selected pavement sections can be modeled.

The purpose of this research effort is to focus on a single input parameter, the elastic modulus of the subgrade. The research is intended to provide insight into the nature of subgrade response as it affects typical pavement sections in Arizona and the Southwest.

Dr. R. A. Jimenez of the University of Arizona is actively engaged in developing a rational method for the design of asphalt pavements in Arizona (1, 2)* based on a three layer elastic model and employing modulus ratios to calculate response. Several other projects are on-going at the Arizona Department of Transportation to determine materials' behavior and environmental conditions under Arizona pavements.

While a final design procedure and pavement management system will be several years in development, it is certain that knowledge of subgrade modulus will be required in three areas.

These are:

- New Pavement Design - The load response of the soil or compacted subgrade is essential in predicting the behavior of any pavement system.
- Evaluation of Existing Pavements - Measured values of elastic modulus for both surface and subgrade layers of existing pavement structures provide excellent indices of deterioration which can be used in planning pavement rehabilitation.
- Pavement Overlay Design Procedures - As is the case with design of new pavements, many agencies are developing overlay procedures which are based on elastic theory. For these, subgrade elastic modulus is an important input parameter.

*Numbers in parentheses correspond to references listed at the end of the report.

The purpose of this project was to develop methods for determining subgrade modulus for Arizona conditions. In the course of this study, the current trends in pavement design were integrated with our experience and a considerable amount of field testing in developing the recommendations contained herein.

1.2 Summary of Program

1.2.1 General

While a broad understanding of the subject of elastic properties of soils is an overall objective of this effort, a more specific goal is to critically examine the load response of typical Arizona soils for which little information is available in the published literature. For this reason, major emphasis was focused on cemented desert clayey sands, sandy clays, and clay-sand-gravel mixtures, and on other partially saturated or desiccated cohesive soils. The body of knowledge available in technical literature concerning the behavior of more classical soil types, such as clean sands and saturated normally consolidated clays, is considered adequate. These will, undoubtedly, receive even more scrutiny from researchers in the next few years and therefore received less attention in this study.

The research effort was divided into five general headings. These are the literature review, sensitivity analysis, field testing, data correlation, and recommended evaluation procedures.

1.2.2 Literature Review

An effort was made to compile a complete bibliography of research related to the subjects of elastic modulus of soils and pavement design. From the various abstract services and other listings, over 400 papers were referenced. More than half of these were obtained and reviewed by the investigators during the course of the project. References will be made to significant findings of many of these research efforts throughout this report.

Specific topics which were researched include:

- Elastic properties of soil and rock, especially as applied to geotechnical engineering or pavement design. Particular attention was devoted to articles concerning materials testing, both in the laboratory and in-place, in order to obtain an understanding of the various ways of quantifying the load response of the materials.

- Relationship between statically and dynamically derived moduli, differences between moduli determined by various tests, effects of size of loaded areas, vertical confining pressure, strain level, and variation in modulus as a function of temperature and moisture content.

- Pavement design methods - every available rational method of pavement design and overlay design which came to our attention, regardless of the degree of refinement, was reviewed. The suggested methods of characterizing subgrade response were of special interest.

- Environmental influences as they affect pavement performance. Studies and case histories of moisture and temperature changes beneath pavements were of interest, particularly those involving pavements in arid regions. An exception within this topic is that of frost related pavement distress. Because of the low rainfall-high temperature environment throughout most of Arizona, the "spring thaw" problem, which is a major concern in many states, was not closely scrutinized.

1.2.3 Sensitivity Analysis

The individual importance of a single input variable within a design system containing many inputs must be measured from the standpoint of economy. In the case of a pavement design procedure, which demands several traffic and materials behavior parameters which are usually not readily available at the beginning of a project, the question of the expense a user may reasonably justify to refine each input value is of major importance.

This report will show that most means of determining moduli are costly in terms of time and money. Therefore, the effect of variations of subgrade modulus on the overall effectiveness of a design procedure must be placed clearly in perspective.

Many sensitivity studies have been performed on specific systems (e.g. 3,4,5) to determine the relative importance of each of the numerous input variables. Rather than duplicate the findings of others, the authors sought a means of presenting the role of subgrade modulus in terms of pavement thicknesses required for various volumes of traffic.

Results of the sensitivity study are given in section 1.9.

1.2.4 Field Testing

A decision was made early in the project to emphasize in-situ testing rather than attempt to test "undisturbed" samples in the laboratory. This decision was made for two reasons.

First, there is a wealth of information in the literature resulting from the dynamic testing of soils in the laboratory, utilizing both undisturbed and reconstituted materials. It was felt that more laboratory data would not add significantly to the existing body of knowledge. Secondly, the Arizona soils of interest are often cemented to varying degrees and are usually jointed or fissured in-place. In many respects, their behavior resembles that of soft rock rather than soil. The taking of "undisturbed" samples in the field is difficult at best and often results in stress-relief which creates further fissuring. Reconstituted samples or samples trimmed for testing are unlikely to duplicate the type of load response which would occur in-situ.

For these reasons, all testing, other than grain-size analysis and Atterberg Limits, were performed in-place at five selected field test stations. The program involved plate bearing tests, refraction seismic, CBR, Dynaflect, and Road Rater testing. Detailed discussions of the test stations and the testing procedures are given in Section II.

1.2.5 Data Correlation

Values of elastic modulus have been calculated for each test. The results of all testing are presented in Section III. In Section IV, an attempt is made to correlate the values derived from the various methods.

1.2.6 Recommended Evaluation Procedures

Recommendations regarding the determination and selection of subgrade elastic modulus values for use in pavement design projects in Arizona, are given in the latter part of Section IV. These procedures make use not only of field and laboratory testing, but also include an evaluation of the likelihood of changes in the subsurface environment during the design life of the project.

1.3 History of the use of Elastic Theory in Soil Mechanics and Pavement Design

In recent years it has become common to find published procedures for analyzing problems of foundation behavior and soil-structure interaction employing models based on elastic theory. To be used effectively, these methods require the integration of two contrasting disciplines; the precise and methodical theory of elasticity and the empirical and interpretative discipline of soil mechanics.

The classical theory of elasticity had it beginnings as early as the 17th Century with the published work of John Hooke. Since that time, the development of elastic theory has been carried on primarily by mathematicians and physicists such as Rankine, Coulomb, Lord Rayleigh, Otto Mohr and others. Until recently, elastic theories were used primarily to describe the behavior of a material whose properties were homogeneous and isotropic, that is, a material which today would be called a "continuum". With the advent of high-speed computers containing large storage capacities, it has been possible to extend elastic analyses to provide solutions to the response of media which are layered or contain other complex geometries.

In contrast, the theories of soil behavior and the art of "soils engineering" have been developed, for the most part, by practitioners during this century and are based largely on empirical evidence. Until recently, soil has been treated as a "particulate" material reflecting the observation that it is composed of discrete grains or particles.

In the early work of pioneers in the field, such as Terzaghi, Casagrande, Bishop and others, major emphasis was placed on the size and gradation of particles, shape of grains, void ratio, friction and cohesion developed by particle contact, and the role of pore water in the determination of soil behavior; particularly for fine grained soils.

During the 1950's and 60's, the state of the art allowed engineers to predict settlement of footings or embankments placed on a given soil based on the results of laboratory tests, such as density, void ratio and consolidation. In the same manner, the load-carrying capacity of a pavement subgrade could be estimated using indices such as CBR or R values.

The worth of any design technique is best evaluated by the performance of the completed product. There have been numerous indications that soil deformations predicted from classical soil mechanics are much greater than actually occur. This may be due largely to the fact that tests, such as the laboratory consolidation test, measure deformation in one direction only while, in reality, deformation takes place in three dimensions.

By 1970, practitioners of soil mechanics, rock mechanics and engineering geology (who were now calling themselves "geotechnical engineers") began to see widespread use of analyses which treated soil and rock as though they were elastic materials. On encountering formulas that demanded a value of E (modulus of elasticity) and ν (Poisson's ratio) for subsurface materials, many engineers simply returned to the more time-honored, but conservative, analyses.

Today, based on extensions of technology developed in the 30's and 40's, the stresses and settlements beneath footings of various shapes can be predicted from elastic analyses using tables or formulas developed by Burmister (6), Giroud (7), Ahlvin and Ulery (8), Poulos (9), Ueshita and Meyerhof (10) and others. Likewise, pile settlements, both individually and in groups, have been treated elastically by Poulos and Davis (11) and Poulos and Mattes (12).

The analyses of lateral capacities of piles and of pile deflections can be made using the elastic approaches developed by Broms (13), Davidson and Gill (14) and Reese and Matlock (15).

During the past two decades, a new methodology has been developed to analyze dynamic response of machine foundations and structures subjected to seismic vibrations. At the low strain levels involved, most material remains in the elastic range; thus, it is no surprise that dynamic analyses of foundations have generated an even greater need for data on the elastic properties of soil and rock. The pioneers in this field have included Hardin, Richart, Seed, Whitman, Lysmer, Hall and Drnevich (see references 16, 17 and 18).

Finally, in the area of pavement design, there always has been a need for some form of stiffness factor for bearing capacity evaluation of the subgrade. Newer pavement design methods which use the computer (such as the CHEVRON and BISTRO Programs) are nearly always based on multilayer elastic theories, as are some of the more simplified longhand methods. Obviously, the results are only as meaningful as the elastic parameters which are used to characterize the subgrade materials.

The physical and mechanical behavior of soils have been found to vary widely with geographical location in such a complex way that subgrade stiffness cannot usually be estimated with sufficient accuracy on the basis of classification and the use of "typical" values. It follows then that an engineer seeking to use elastic analyses for projects in his area, should not be content to pick values out of a handbook, but should investigate the meaning of the term as applied to his design situation.

1.4 Available Rational Pavement Design Methods

1.4.1 General

In most geographical provinces, the final responsibility for design of highway and airfield pavements resides in some division of a Department or Ministry of Transportation. In the case of highway pavements, construction usually proceeds on a continuing basis from year to year, and design techniques tend to become established routines.

Those engineers whose job it is to actually design pavement sections are likely to be confined by departmental directives to a structured, step by step procedure. The method being used at a given agency may have been developed originally by or for that agency, or it may have been borrowed (possibly from afar) and modified for local use.

Pavement technology is usually divided into two general categories, that concerned with flexible (asphaltic concrete or bituminous bound) and rigid (Portland Cement concrete) pavements. Hybrid structures, such as those involving stabilizing bases, lime-cement-fly ash sections, or bituminous overlays of concrete pavements often do not fit neatly into either category.

Regardless of the materials comprising a pavement, the characteristics of the subgrade are important. For conventional flexible pavements, the compacted subgrade, along with the aggregate base, is an essential structured element. In the case of rigid pavements, deflections, moments and shears are functions of interaction with the subgrade which acts as a foundation.

Many more sophisticated design procedures have been published in recent years, particularly for flexible highway pavements. These methods are the result of research by universities, industry, government agencies and private engineering consultants, or in most cases, cooperative efforts involving two or more of these entities. The published methods available in 1977 vary greatly in their degrees of sophistication, stages of development and particularly in their relative amounts of field verification. It is almost certain that as more agencies take careful note of their needs in the area of pavements, the number and complexity of design systems will grow.

While it is not the purpose of this effort to judge the merits of any design procedure, a brief mention of many methods is presented. A further look at these methods, with respect to the ways the various researchers recommend characterizing subgrade response is presented in section 1.8.

1.4.2 Rigid Airfield Pavements

Because of the nature of airfield construction, designs are more likely to be pursued on an individual basis, rather than on a production basis, as is often the case with highways. Factors such as the multiplicity of aircraft types and loading, relative uniformity of subsurface conditions over the length of construction, and intense interest in the project on the part of a handful of intended users, dictate the more individual approval. There are only a few published methodologies, four of which are noted here.

A. Portland Cement Association Method (Reference 19)
   This thickness design method, presented in chart form, is easy to follow. For several specified aircraft, thickness requirements are presented in terms of repetitions of load to failure for various stiffnesses of subgrade. Subprocedures are available to analyze mixed load of traffic.

B. Corps of Engineers Method (Reference 20)

This method is similar to the PCA method, making use of charts for light, medium, and heavy aircraft. A subsystem for the reduction in subgrade strength during frost action is a major feature.

C. FAA Method (Reference 21)

The charts prepared by the PCA were incorporated into computer programs employing more sophisticated traffic analysis subsystems. Thickness requirements for single gear and dual-tandum gear aircraft are given in terms of a 20-year, 20,000 pass life.

D. Yang Method (Reference 22)

In his textbook, Yang describes an intricate deterministic analysis approach to the design of airport pavements. The subsystems are based on experience gained during the rebuilding of Newark Airport by the Port of New York Authority.

Some of the many subsystems included in the method are, vehicle-pavement interaction models, progressive deterioration models and cost-benefit (fiscal management) models. Heavy emphasis is placed on dynamic testing of materials. While the textbook example is valid only for Newark, Yang states that the models can be tailored and input parameters obtained for other airports. Obviously, the use of this method, or any of its subsystems, can be justified only on major projects requiring in-depth analyses.

The Yang method is one example, of which there are several in the highway pavement area, where the models are highly advanced and seemingly all possible factors related to pavement performance are included. Questions arise, however, as to field verification of the procedure. Also, there arises in the minds of many practical engineers, the suspicion that our ability to conceive and formulate performance models may have outdistanced our ability to provide meaningful input parameters.

1.4.3 Flexible Airfield Pavements

The design methods for flexible airfield pavements are largely empirical and have been in use for many years. As with rigid pavements, a key element is the design aircraft, its wheel load configuration and the traffic mix. Three methods are referenced:

A. Corps of Engineers Method (Reference 23)
This empirical method was developed during World War II and is based on CBR. Modifications have since been made to accommodate the heavier wheel loadings. The method is presented in the form of charts which allow rapid thickness evaluation for different loadings and traffic patterns.

B. FAA Method (Reference 24)
This method also uses CBR and is given in the form of charts which present base thickness and

bitumen thicknesses required for single and dual wheel gear loads.

C. Asphalt Institute Method (Reference 25)
This is a "full depth" asphalt design procedure. It is similar to the Asphalt Institute's method for the design of highway pavements, but special attention is given to the problem of mixed traffic loadings at airfields. Subsystems dealing with the fatigue and deformation properties of asphalt are a major feature. Thus, the Asphalt Institute method is more deterministic than the other two methods.

1.4.4 Rigid Highway Pavements

The economics of construction normally dictate that rigid highway pavements be placed where roads must carry a high volume of heavy traffic. The performance of rigid slabs is dependent to a large degree upon the strength of the concrete and to a somewhat lesser degree on the strength of subgrades.

Probably because of the low pavement mileage built of PCC (compared to flexible pavement mileage), few agencies have developed design methods of their own. The two methods given here (PCA & AASHTO) appear to be sound theoretically and apparently few problems have arisen with their use.

A. Portland Cement Association Method (Reference 26)
The Westergaard stress analysis technique is the

basis for this method. Traffic is divided into load axle groups with provisions incorporated to allow for traffic growth in the future. The concrete strength and thicknesses required for an anticipated traffic load over a 40-year life, can be determined using a tabular procedure.

B. AASHTO Method (Reference 27)
This easy-to-use method was developed in the 1960's, based on results of the AASHO road test in Illinois. Utilizing a set of nomograms, it is a simple procedure to determine a necessary slab thickness based on conditions of traffic, concrete strength and subgrade support for a 20-year life.

1.4.5 Flexible Highway Pavements

Highway pavements utilizing bituminous bound layers, either as structural units or for the purpose of sealing and smoothing the riding surface, are, by far, the most abundant in terms of mileage throughout the world. Accordingly, these have received the most attention of researchers. For many years, empirical techniques and local engineering judgment formed the basis for most designs. Since the 1950's, there has been an increasing effort to replace empiricism with rational methods in the design of flexible pavements.

At the time of this writing, the Fourth International Conference - Structural Design of Asphalt

Pavements, has just been completed. This international gathering of pavement specialists has been held at five year intervals since 1962. At this recent conference, emphasis was placed on complete (or nearly complete) pavement design systems. Methods were presented from every corner of the developed world.

Compared to the pavement types mentioned previously, the number and degree of sophistication of design systems and subsystems dealing with flexible highway pavements seems overwhelming. This abundance of technology is partly due to the universal acceptance of asphalt pavements, but there are other factors. Among them are:

- The constituents and behavioral properties of bituminous mixes cover a very broad range.

- There are several manifestations of distress and failure in asphaltic concrete pavements (fatigue cracking, rutting, low temperature cracking, etc.). Most means of quantifying states of pavement deterioration are, to varying degrees, subjective.

- Some forms of distress are not a function of traffic, but result from aging or environmental attack on the bituminous materials. In some regions, these are important considerations.

- Climate and subsurface environmental conditions vary widely from one location to another.

• The economics of construction (and rehabilitation) are a function of geography and are subject to change with time.

• Traffic loads on asphalt pavements range from a few vehicles a day to many thousands.

• Legal load configuration and tire pressures vary greatly from country-to-country.

At the recent international conference, there was a debate among the practitioners over whether design methods were becoming so complex as to be cumbersome for the average engineer to understand and use. A plea was made that methods must be simplified by the pavement specialists, in order to be accepted by the general engineering community.

Those who advocate sophistication urged complete understanding of all mechanicisms as a basis for future simplification and development of workable techniques. Those who argued for simplification first, stated that few organizations were rushing to take advantage of these new tools. It was noted that there is little benefit in developing even more complicated and expensive methods of analysis, because if they are not widely used, their accuracy will never be verified.

Nevertheless, several of the methods deserve brief mention here and in section 1.8, as they will undoubtedly be in the forefront of pavement design technology. The first three (VESYS IIM, PDMAP, and Shell) are the most inclusive as of this date. Two others (Chevron Research and Kentucky) are in a form which can be easily used by practicing engineers and were employed in our sensitivity study (section 1.9).

A. VESYS IIM (Reference 28)

The VESYS IIM system is probably the most advanced and all-inclusive flexible pavement design method available today. It has been formulated within the U. S. Department of Transportation-Federal Highway Administration in cooperation with engineers from the Massachusetts Institute of Technology and Austin Research Engineers. The most striking features of VESYS are the models used for materials characterization, particularly the creep-compliance curves which are necessary to describe visco-elastic behavior. Employing the multitude of inputs supplied by the design engineer, computer programs calculate the anticipated performance in terms of rutting, roughness and crack damage.

The method is being implemented on an experimental basis by the Departments of Transportation in Utah and Florida for the purpose of verifying and refining the many subsystems and improving the method as a whole. VESYS IIM is understood in depth by only a few engineers, therefore, its adoption by existing agencies is likely to be slow. One of the biggest problems is said to be the collection and characterization of field and laboratory data in exactly the way the developers of the various models intended.

B. PDMAP (Reference 29)

A sophisticated computer program has been developed by Woodward-Clyde consultants of San Francisco for the analysis of fatigue and permanent deformations of flexible pavements. Since this firm is a consultant to Arizona Department of Transportation in the development of a pavement management system, the PDMAP procedure is of special importance to Arizona. As is true of most of the newer systems, models to characterize materials behavior are an important feature. The developers admit there are problems with field verification, especially in the area of damage criteria for the fatigue model.

PDMAP has been used on field test information from California and Minnesota where verification continues. Closely allied with PDMAP is another computer model, called COLD, for analyzing low temperature cracking.

C. Shell Method (Reference 30)

Many years of quality research in pavement and asphalt technology at the Koninklijke/Shell Laboratorium in Amsterdam, and elsewhere on the European continent, have resulted in a well documented Shell Method for the design of flexible pavements. At this time, the method has been refined to the point where it is inclusive, but in a form that can be used without difficulty by most engineers. The method has been reduced to charts, making computers unnecessary. Like all methods, there is a need for further verification of the systems; which is continuously being performed in the field.

D. Kentucky Method (Reference 5)
Probably the most advanced and inclusive procedure developed completely in-house by a State Department of Transportation, is the Kentucky Flexible Pavement Design and Management System. The procedure and its subsystems are available in chart and graph forms. They include models for analyzing fatigue and rutting deterioration of pavements, traffic mixes and economic analyses.

The Kentucky Highway Department has performed several sensitivity studies of their own models and have provided considerable field verification for the method. While it is tailored specifically for use in Kentucky and, therefore, some subsystems may not be applicable in other regions, the procedures are well worth considering.

E. Chevron Research Method (Reference 31)
Probably the simplest method to use while still incorporating sophistication, is the Chevron Research Method. Charts have been prepared to

determine thickness requirements for full depth asphalt pavements based on rutting and fatigue. The major emphasis in the subsystems concerns the properties of various asphalt mixes.

F. Other Design Procedures

Many other pavement design procedures have been advanced here and abroad. The most widely used is the AASHTO method (27). This procedure, which makes use of layer equivalency ratios, has been adopted in some form by many of the state Departments of Transportation. The Asphalt Institute (32) has a widely used method that is still being modified. The National Crushed Stone Association (33) has developed, in cooperation with the Corps of Engineers, a straightforward method which is largely empirical.

Several state Departments of Transportation have published flexible design methods. Notable among these are; Ohio, Kansas and California. The Texas Transportation Institute, in cooperation with independent consultants and university researchers, has published numerous excellent papers dealing with pavement design subsystems.

Research in the area of flexible pavements is increasing abroad also. In Canada, researchers at the Department of Transportation (CDOT) and the University of Waterloo have contributed substantial knowledge. In the United Kingdom, the work being done at the Road Research Laboratory, Crowthorne, and the University of Nottingham, deserves special note. Other pavement design methods, some very sophisticated, have come from France, Germany, Belgium, Ireland, Denmark, Italy, Hungary, the USSR, South Africa, and Australia.

It is not within the scope of this project to perform more than a general evaluation of the various pavement design procedures or subsystems. Our main interest has been in the various conventions which pavement experts from throughout the world have chosen or recommended to characterize subgrade response.

In most cases the subgrade (soil), as well as other materials in the pavement prism, has been modeled elastically. On the surface this appears to combine sophistication with simplicity. But soil does not behave elastically in most situations and therefore an elastic response model is an approximation of what really occurs. For this reason, there have evolved many ways of defining and measuring soil response for incorporation into design procedures. A look at the current state-of-the-art concerning elastic properties of soils is presented in the following section.

1.5 The Concept of Modulus as Applied to Soils

1.5.1 General

In classical theory of elasticity, the modulus of elasticity (or Young's modulus) of a material is that numerical value which expresses the proportionality of stress to strain. Graphically, it is the slope of a stress versus strain relationship.

An examination of a typical stress to strain plot, such as given in Figure 1, will show that a single unique value of Young's modulus can be obtained only if the stress-strain relationship is linear. Otherwise, the elastic modulus must be expressed as a function of stress level.

In the real world, few earth materials exhibit a linear stress-strain relationship through a significant range of stresses. It is probably a very rare soil or soft rock which displays a linear response to loading except at very low strain levels.

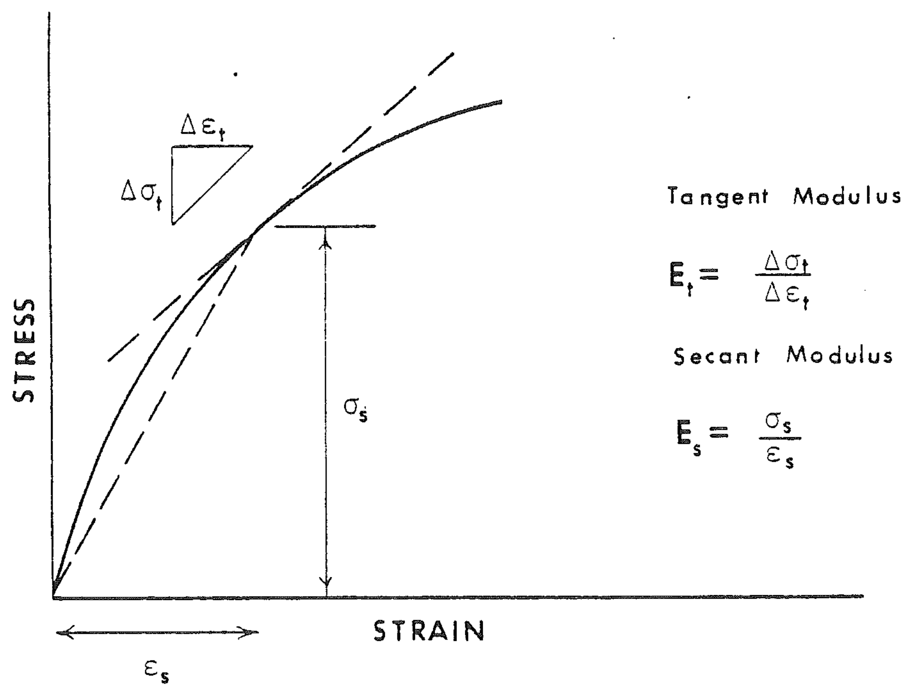


Figure 1. Graphical Representation of Elastic Modulus

Engineering formulas which make use of an elastic model demand a single number to describe what is, in essence, the "response" of the material to being loaded. In most cases, this toys with reality. The response of the material is a function of stress level, f(p). In fact, as will be shown in later sections, the load response is a function of many more than one variable. In addition to the magnitude of stress, the response may be dependent upon the size and shape of the loaded area, the thickness(es) of the loaded layer(s), and the relative stiffness of the various materials in the layered system. In designing a facility, such as a paved roadway, the possibility that the response will change with time or with seasonal variations in environmental factors (i.e., moisture content, temperature) must also be considered.

In the early studies which gave rise to the discipline known as soil mechanics, the three phase (solids, water, and air) component system of soil, along with the particulate nature of the solid component, was recognized and examined. These unique characteristics of soil made necessary an entirely new methodology to describe its mechanical behavior. As opposed to metal, wood, brick, or stone, which could be considered a "continuum", soil was labeled a "discontinuum". Experiments have shown that most materials which are continuums are adaptable to theories of elasticity or plasticity at least through some stress range. However, one may properly question whether a particulate system, such as soil, is receptive to elastic analyses, especially when subjected to tensile stresses.

The consensus among active researchers in the profession today seems to be that on a macro scale, soil and earth materials do indeed exhibit behavior similar to that of a continuum under many conditions of loading. In contrast, rock, which was assumed to be a continuum and highly adaptable to elastic theory, has been found to behave on a macro scale more as a discontinuum. The engineering behavior of rock in road cuts or large excavations is governed by the natural planes of weakness, such as bedding planes, joints and fissures, rather than by the strength properties of the rock substance.

Returning to the response of earth materials to load, one can see there is a definite anomaly. On the one hand, the engineer seeks to use design techniques which require that the response of the soil to load be expressed as a single numerical parameter much as one should insert a value of bulk density or specific gravity of solids into a formula.

Yet, in reality, the response (hence modulus) is not an intrinsic property, but a highly dependent variable. The engineer knows that he is using expressions which were developed for a "continuum", yet, intuitively, he senses that his three phase particulate system is somewhat different.

On the other hand, engineering is an art which, throughout history, has merged scientific doctrines with empirical evidence to develop sound and beneficial design procedures. An objective of this study is to seek such an accommodation as can be applied to pavement design in Arizona.

In technical literature, one often sees the term "modulus" (usually in conjunction with a modifier) used rather casually and denoted by any of a myriad of subscripted alphanumeric characters (e.g., E_z, E_o, N_x, D, M_R, G). The burdening variety of names and notations leads the novice to wonder whether or not the authors are talking about the same thing. In general, they are. All of the "modulus" terms are measures of response of a medium or layered system to stress. The confusing multiplicity of terms is due to the many ways in which response can be induced, charted and quantified. Specific engineering analyses require parameters which quantify the likely behavior of materials under a stress system similar to that to which the material will be subjected during its design life. For instance, an analysis of the foundation for a sensitive piece of vibrating machinery requires quantification of soil response under that peculiar type of loading. A long slender pile carrying a lateral load in the same soil would generate an entirely different response from the soil.

In this chapter, some of the more common moduli will be discussed, although the treatment is not intended to (nor could it) touch on every response term one may encounter in the literature.

1.5.2 Compression Modulus (E)

In continuum mechanics, the most commonly used response parameter is simply the ratio of the induced stress to resultant strain termed the "modulus of elasticity", or "Young's modulus", and is universally designated by the character "E". In the literature of soil mechanics, the expression "Young's modulus of the soil" is occasionally found, but because soil is not truly elastic, many writers prefer a substitute term. Modulus of elasticity and compression modulus are most commonly used. These carry the designator E, thus affirming that the parameter is rooted in elastic theory. Farther afield in connotation, though not in physical meaning, is the term "modulus of deformation", sometimes designated as M, which seems to be in little favor of late. The wider acceptance of elastic analyses in soil engineering in recent years seems to have eliminated some of the artificial distinctions of terminology, and there now appears to be no need to apologize for referring to E as Young's modulus of soil.

The compression modulus (E) is expressed in units of stress; strain being dimensionless. In the English system, values are usually given in psi.

Ideally, the E modulus defines the response of an earth material loaded over any area (or loaded at a point); that is, shape and size of the loaded area are not of consequence in the definition. In reality, any load must be applied and measured over a specific area. The soil whose response is measured may rest under a footing, a plate or a beam in the case of field tests, or it may be a sample of finite dimensions in the laboratory. In the literature there are many correlation equations which seek to eliminate boundary effects in an attempt to provide a more nearly intrinsic property to be called "compression modulus".

An example is the work published by McLeod (34) of the Canadian Department of Transportation concerning the influence of plate size on the unit load causing a given deflection. Another is an empirical equation given by Vesić (35), which will be discussed further in section 1.5.3.

In all such studies, correlations are probably applicable only to the particular soil types which are the object of a given investigation. Universal applicability of such relationships is unlikely; therefore, the need arises to provide a standard compression modulus at a selected stress level and bearing pressure.

Because of the inherent ambiguity surrounding modulus of elasticity, with regard to soil, the character E is often found to be subscripted in the literature. This in itself leads to more confusion, but it is necessary to alert the user to some special constraint which may be important in the use of the parameter.

The most commonly seen subscript is E_s. Unfortunately, the writer knows of three contexts in which this subscript has been used. The most common application of E_s is to denote that the modulus was computed as a secant modulus (as opposed to a tangent modulus; see Figure 1. That is, the response is a measure of total strain, rather than a ratio of incremental values.

In the soil mechanics literature, secant values of modulus are by far the more common and, in this study, E modulus values will be assumed to be secant values except in the regime of very low strains where the initial tangent modulus and secant modulus are virtually synonymous.

E_s is also used (especially in the literature concerning pavements) to specify the material in question is native soil or subgrade, as opposed to the man-made layers in the pavement systems. Moduli of pavement and base course layers are usually designated E_1 and E_2, respectively, and subgrade modulus is E_3 in a three layer system. In this study, the preceding definition of E_s will apply.

A third type of modulus, which sometimes receives the designation E_s, is the seismic or dynamic modulus. This also has other designators as will be discussed later in this chapter.

Many other subscripts (and superscripts) which are found attached to the E modulus in the literature have meaning only as defined in that specific treatise. In another author's work, the same subscript may be used for an entirely different purpose. A few general examples taken at random from the literature include E_o for the modulus of soil in a completely dry state, E_n for the modulus determined in the unconfined compression test, E_c for the modulus determined in the consolidation test (this probably is the same as D, the constrained modulus to be discussed later) and E_p for the modulus determined using the Menard Pressuremeter. In one respected text, E_f refers to the soil modulus applied to a loaded footing, while in another, E_f is the secant modulus at failure of a triaxially loaded specimen.

While the means of quantifying a stress-strain relationship is essentially the same for all the many subscripted variations of E, the numerical values which are obtained by testing a given material usually vary widely as will be seen in sections 1.6 and 1.7.

1.5.3 Modulus of Subgrade Reaction

Perhaps the oldest, and most widely used, response parameter in soil mechanics is the modulus of subgrade reaction which is nearly always designated k. This parameter has been used for many years in evaluating subgrades for rigid pavement design, and variations of k (namely k_h and its derivative n_h) are very important in the analysis of laterally loaded piles.

Physically, the modulus of subgrade reaction is the ratio of stress to deflection upon a loaded area. Its units of measurement are stress divided by deflection, usually expressed in psi per inch of deflection or pci.

The k modulus was first described by Westergaard, and was called the "dense liquid constant". This name invokes the idea of soil being modeled as a dense liquid, rather than an elastic solid. Presumably then, the soil deforms without volume change.

In more recent writings, k is referred to as the spring constant of the soil. Such terminology envisions a model in which a rigid body transmits load to a flexible soil whose response would mathematically be the same as the spring in the classical Hooke model. In the context of the subject of rheology, where the response of a material is modeled in terms of a spring, sliding mass, and dashpot, the use of k implies a purely elastic system. A visco-elastic system would require a spring constant and a dashpot constant, while an elasto-plastic system would require spring and sliding block constants.

In the early works after Terzaghi, the word "coefficient" was used rather than modulus in connection with k. This has lead to confusion because, in some papers concerned with piles, "coefficient of subgrade reaction" and "constant of subgrade reaction" are used for another purpose. These are discussed later in this chapter.

The k modulus is similar to the E modulus in many respects, and one might successfully argue that they are, in reality, the same thing. The difference lies more in their connotation and use than in what one physically measures in the field.

Historically, k is measured in the field using plate bearing tests. There has been no attempt until recently to tie k to elastic theory. There has always been a recognition that the size and shape of a loaded area strongly influences the k value measured and, as is the case with E, expressions of correlation have been proposed. An example is Terzaghi's (36) expression $k_m = k\left(\frac{m + 0.5}{1.5\ m}\right)$, where k is the modulus of subgrade reaction for a square plate of dimension B and k_m is the modulus for a rectangular footing with dimensions B x mB. Another Terzaghi equation relating modulus to plate size is, $k_s = k\left(\frac{B + 1}{2\ B}\right)^2$ where k is the modulus determined from a 1 foot square plate and k_s is the appropriate modulus for an area of dimension B.

The above equations apply only to sands. For clays, the relationship $k_s = \frac{k}{B}$ is given. Vesić (35) has given an expression for correlating k modulus with footing size based on E_s (Young's modulus of the soil). The equation is: $k'_s = 0.65 \sqrt[12]{\frac{E_s\ B^4}{E_b\ I}} \quad \frac{E_s}{1-\nu^2}$

where B is the footing width E_b is the elastic modulus of the footing and I is the moment of inertia of the footing. Also, $k'_s = k_sB$. This equation seems to indiate that Vesić believes the E modulus to be a much more fundamental parameter of soil than the k modulus.

The problem with all of the above correlation equations is that they probably are valid only for ideal materials. The degree of error (and the consequences) which may result from applying these relationships to a specific earth material will probably remain unknown.

Pursuant to a discussion of modulus of subgrade reaction (k), it is appropriate to discuss briefly the relatively similar terminology used for load-deflection analyses of laterally loaded piles (or soil-pole interaction theory as it is sometimes called). As in other analyses, the concept of (k_x or k_h) as a spring constant of the soil is used. However, since large depths are involved, usually an increase in stiffness of the soil with increased vertical confining pressure must be included in the analysis. For this reason, Terzaghi in 1955 introduced a term called "constant of horizontal subgrade reaction" designated n_h. This term could be applied to those soils (especially sands) whose stiffness increases linearly with depth, i.e., $k_x = k_o + n_hx$; x being depth below the ground surface.

Since Terzaghi's first writings on the subject, there has been no standardization of notation or terminology in the later works on soil-pole interaction theory, and confusion is the order of the day. Researchers such as Broms (37) and Davidson and Gill (38) have generally followed Terzaghi's (39) conventions (with some additions of their own) but Matlock and Reese (40), who have published voluminous research on this and related subjects, use k for the derivative of the subgrade reaction modulus with respect to depth instead of n_h and use E_s as a spring constant rather than k. In a recent work in the field of load-deflection analysis of piles by Poulos (41, 42), another change in notation and a shift closer to pure elastic analysis can be noted. Poulos' soil stiffness is given in terms of Young's modulus (E) rather than k and n_h is the derivative used to denote the variation of E with depth. Spring constants (k) are not used except in general discussion.

Dimensionally, the terminology of soil-pole interaction theory provides confusion when compared with similar terms from other areas of geotechnical engineering. One would expect the first derivative of the spring constant to be given in units of pounds per $inch^4$, and a few writers have used this. Most, however, such as Reese and Matlock, prefer to express deflections and spring constants as per unit width of pile. This results in constants of subgrade reaction being given in terms of pounds per $inch^3$ and spring constants in pounds per $inch^2$.

It should be evident that the terminology and conventions of soil-pole interaction analyses, while similar to those of other areas of geotechnical engineering, are unique and should be used only in their own context. An excellent summary of the subject of horizontal subgrade reaction has recently been published by Audibert and Nyman (43).

In conclusion, it is the writer's opinion that in future treatises dealing with foundations, piles or layered pavement systems, modulus of subgrade reaction (k) will increasingly give way to Young's modulus (E). As more sophisticated analyses, especially those dependent upon computers such as finite element, require E moduli to describe the soil's response to load, engineers are becoming more comfortable using a Young's modulus for soil. Apparently a major reason for the increasing popularity of E versus k lies in the belief that E is dependent upon fewer variables than k and, thus, is slightly closer to being an intrinsic property of a given earth material.

1.5.4 Shear Modulus (G)

The shear modulus of soil designated G (and sometimes called modulus of shear deformation) is a useful parameter because of its precise derivation from elastic and vibration theory. Within the range of low strains, G comes very close to being an intrinsic property of most earth materials. Two equations widely used in soil dynamics describe fundamental relationships.

The first defines shear modulus $G = \rho V_s^2$ where ρ is the mass density and V_s is the velocity of shear wave travel through the material. Most engineers can agree that ρ and V_s are intrinsic properties of a given earth material that can be determined definitively through field or laboratory testing (though with great difficulty in the case of shear wave velocity).

A second equation relates shear modulus to compression modulus.

$$G = \frac{E}{2\ (1+\nu)}$$

It would appear at first glance that this equation provides the means to define E intrinsically as well. One must remember, however, that the shear modulus is measured as elastic vibrations travel through a medium, the shear wave being only one of many waves. Most of the field geophysical techniques in use today produce unit strains which are usually on the order of 10^{-4} to 10^{-6} inches per inch. Thus, the shear modulus and the Young's modulus derived in such a manner are applicable only to that regime of low strains, which for most earth materials is considerably different than the response under a plate or foundation where several tenths of inches of deflection may be involved.

As a possible remedy to the problem of strain level dependence, engineers from Shannon and Wilson (44) have sought to develop an in-situ impulse test. Waves are generated by the free fall of a heavy hammer on a specially designed anchor which is placed in a borehole. They have measured shear wave velocity at strain levels as high as 3×10^{-3} inches per inch, which is close to the strain produced by strong motion earthquakes. Correlations of shear modulus with elastic modulus at even higher strain levels are needed, however.

For sands, Hardin and Drnevich (45) and Seed and Idriss (46) recommend that G modulus be determined in the field or laboratory using shear wave velocity at the low strain levels, and then reduced by some empirical method to obtain reasonable values of modulus at other strain levels. For saturated clays, G moduli values determined by laboratory and by field techniques were found to be farther apart, yet there seems to be no reason why modulus reduction techniques similar to those used for sands cannot be developed for soils of all types.

1.5.5 Dynamic Modulus

Inasmuch as a large part of the recent research in the field of load response of soils makes use of vibration techniques, it is not surprising to see response parameters with designations such as "dynamic modulus". Unfortunately, the exact meaning and characterization of the term varies widely.

For example, it was previously noted that E_s is used in some works to denote a Young's modulus determined seismically (presumably at low strain levels). In other works, dynamic modulus and shear modulus (G) are used interchangeably. Recent articles by Hall

(47) and Weiss (48) of the U. S. Army Corps of Engineers Waterways Experiment Station use a parameter called the dynamic stiffness modulus (DSM) in the methodology applied to their nondestructive testing of pavements. The DSM is a spring constant and must be determined by field testing as is the subgrade reaction modulus (k).

In essence, dynamic moduli are the same as the response parameters discussed earlier in this chapter. The distinction arises from the type of loading and the range of strains through which they apply.

1.5.6 Resilient Modulus (M_R)

Modulus of resilience (M_R), also called resilient modulus of deformation, is an index of the response of materials to repeated load and is primarily used in the field of pavement design. A specially designed triaxial testing cell is necessary to perform the test. Response properties of both bituminous bound and unbound materials may be determined in this manner.

Resilient modulus is defined as the ratio of deviatoric stress to recoverable strain. Figure 2 provides a graphical presentation of its meaning.

$$M_R = \frac{\sigma_d}{\varepsilon_{rec.}}$$

To be meaningful in design, the value of M_R must be related to a particular value of deviator stress and number of cycles of loading.

The laboratory test is difficult and expensive to run due to the care necessary in sample preparation. There has been increasing interest in the recent literature concerning M_R. Many pavement analysts treat M_R as an appropriate substitute for E to describe the load response of soil, especially a soil subjected to cyclic loading. More will be said concerning triaxial testing in section 1.6.4.

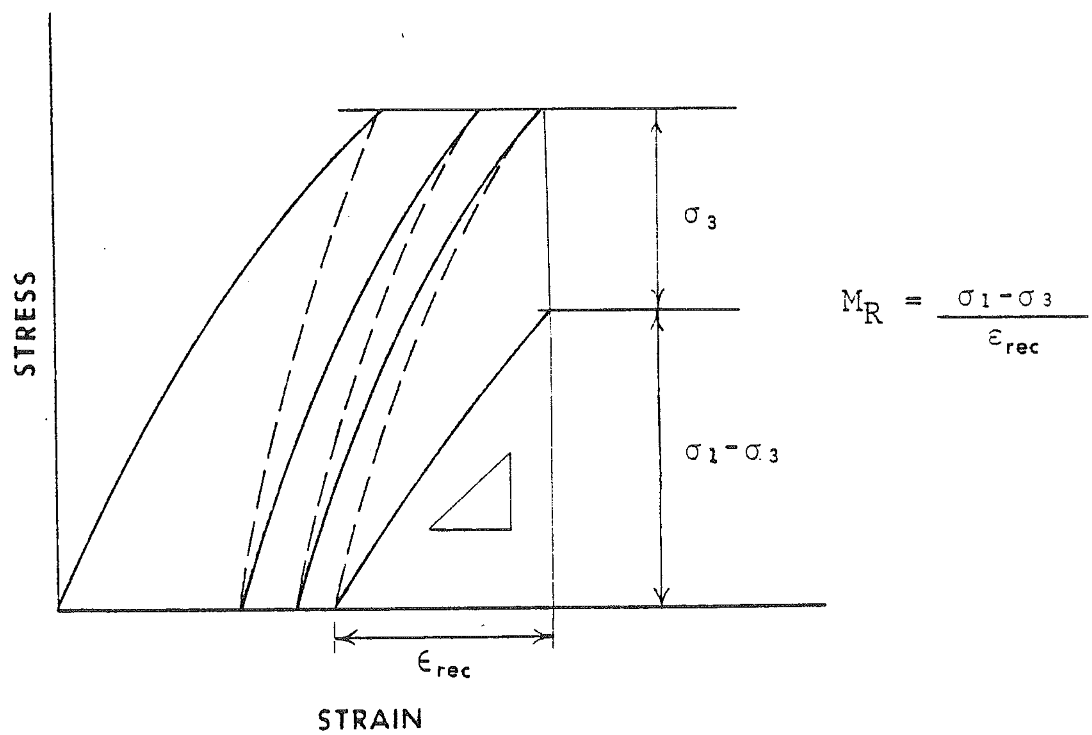


Figure 2. Graphical Presentation of Resilient Modulus

1.5.7 Confined Modulus

Confined or constrained modulus usually refers to the compression modulus derived from the one-dimensional consolidation test, or any test in which volume change is permitted in one direction only. This modulus receives various designations in the literature, most commonly E_c or D. It is very important for the engineer to recognize that since real world loading situations result in volume change in three directions, the constrained modulus must be corrected. From elastic theory comes the relationship

$$D = \frac{E\ (1-\nu)}{(1+\nu)\ (1-2\nu)}$$

By inspection, it can be seen that the conversion is heavily dependent upon the value of Poisson's ratio; thus, the engineer desiring to use this expression must have a feel for the ν of his material.

1.5.8 Bulk Modulus

Occasionally in the soil mechanics literature, bulk modulus (B) or (K), also called modulus of volume compressibility, is found. This term is an index of percent change in volume of a material in response to a given load. It is defined as $B = \sigma/\Delta v/v$. Where volumetric strain is equal in all three directions, elastic theory provides that $B = \frac{E}{3\ (1-2\nu)}$. Percent change in volume can be measured only on a sample of finite dimensions and the equalization of strains in all directions is most difficult to achieve. For these reasons, bulk modulus is seldom used in engineering analysis.

1.5.9 Complex Modulus

Complex modulus $|E^*|$ is a special term used in asphalt technology to describe the stress-strain relationship of a visco-elastic material under simulated traffic in the laboratory. Its use is described by Papazian (49). Essentially, complex modulus compares maximum stress to maximum strain, which in a pulse or sinusoidal load situation may not occur at the same instant. Apparently, complex modulus is not used in connection with earth materials.

1.6 Determination of Modulus in the Laboratory

1.6.1 General

Virtually any test in which load and deformation are measured can provide a means of calculating a value of elastic modulus of the soil. Unconfined compression, uniaxial compression, plane strain and triaxial testing are well known in soil mechanics, and are often used to determine and compare modulus values.

What the engineer is attempting to accomplish is to solve a boundary value problem in soil mechanics (in this case the layered pavement) by inserting a laboratory determined parameter which hopefully simulates in-place soil behavior. On large jobs involving great distances, such as highway pavements, the production conscious engineer would prefer to use the simplest procedure which yields acceptable results. Several comparison studies have been performed whose objectives were to determine modulus for a particular soil type by several methods and compare them with field experience. A few of these studies merit brief mention.

Forsyth, et al (50) from the California Department of Transportation, investigated the response of a stiff residual clay (actually a weathered shale). Moduli were back calculated from plate bearing tests and then also determined by means of unconfined compression, triaxial, consolidation, Menard Pressuremeter and seismic techniques. Unconfined compression and triaxial tests resulted in moduli too low, probably because of sample disturbance. The other test methods produced a wide range of stress dependent values.

Krizek and Corotis (51), investigated the response of well graded sand-gravel and sandy silt mixtures in laboratory tests and compared them with the response of compacted fills of the same material. Values obtained from uniaxial compression, plane strain, conventional triaxial and true triaxial tests showed that modulus was highly dependent on the stress path and type of test. For the compacted granular soils, dry density and stress level were the most important variables affecting modulus.

Shields and Bauer (52), investigated the response of a sensitive clay both in the laboratory and in-situ. They found that for this particular soil, there are two ranges of E_s values. An upper range of values was obtained from consolidation, triaxial and plate bearing tests; a lower range from unconfined compression and pressuremeter tests. These results contrast with those of Forsyth, although it is virtually impossible to compare results of testing involving different laboratories and different soils.

Thompson and Robnett (53), sampled fine grained soils from sites throughout the State of Illinois. Attempts were made to correlate resilient modulus from the triaxial test with soil classification and CBR tests. In their initial report, correlations were found to be not completely satisfactory.

It has been found by virtually every investigator who has undertaken the task of studying the load response of earth materials, that modulus values must be qualified as to the conditions of the test and the means of interpreting the data.

1.6.2 Classification Tests

There is a natural tendency for engineers to measure those properties of materials which are easy to measure in order to obtain a maximum amount of information for a given effort. Soil classification tests, dependent on grain-size distribution and plasticity index, are part of nearly all soil investigations.

This is partly due to the usefulness of these parameters, but also because they can be scheduled for high volume production.

It is not surprising, therefore, that many designers seek a correlation between Unified, AASHTO, or FAA soil classifications and elastic modulus. An example of such correlations is the graph given in Figure 3, which has been published by the Portland Cement Association. It can be immediately seen that the correlations allow wide latitude in interpretation. They reflect the general observation that coarse granular soils, such as SW, GW and GP, are stronger than CL, ML and CH soils.

Such correlation charts may be acceptable where very imprecise and conservative designs are to be used, but it should be obvious that they are fundamentally inadequate. We have previously stated that modulus of soil is a function of many variables, such as stress level, size of loaded area, and particularly, moisture content. In no way does a classification test account for these. This is particularly true in Arizona where naturally occurring lime cementation is often present, acting to strengthen dry soil. Classification tests have no inherent way of accounting for the stiffness of such soils.

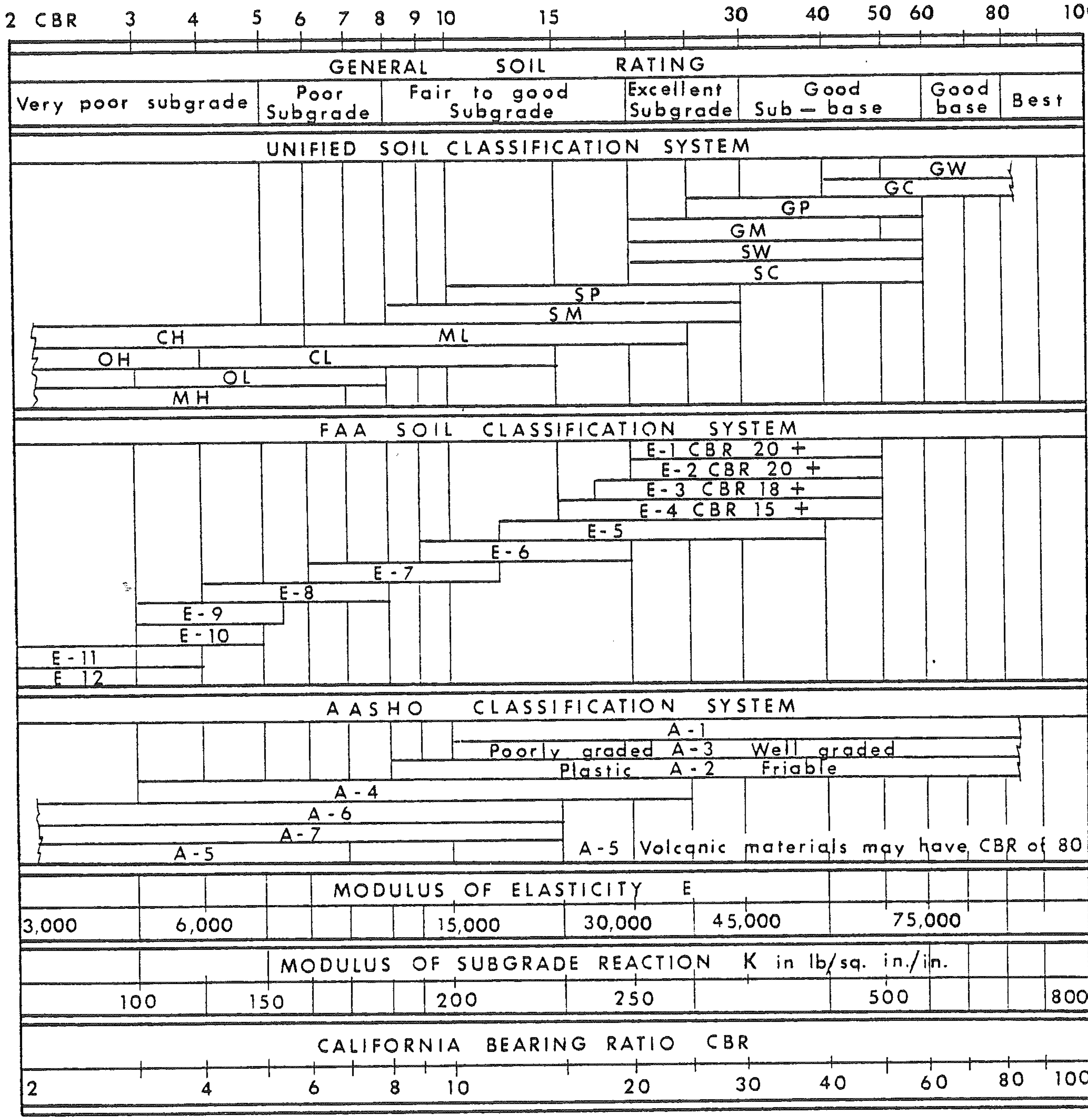
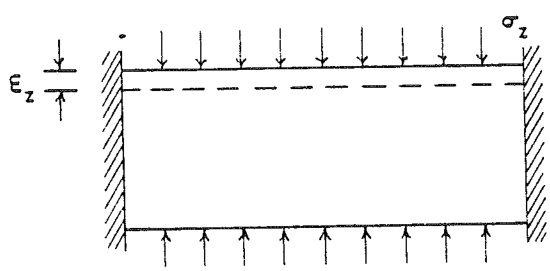


Figure 3. Correlation of Modulus Values with Soil Classification (after Reference 19)

Attempts have also been made to correlate modulus with relative density or void ratio in granular materials. An example is Figure 4, taken from a textbook by Richart, Hall and Woods (54). The problem with such a graph is that it applies only to very dry sand, probably only to uniformly graded sands, and is applicable only at very low levels of strain. This last problem can be partly overcome by applying strain-reduction factors such as given by Seed and Idriss (46) in Figure 5.

1.6.3 Uniaxial Compression Tests

As mentioned in section 1.5, the confined modulus D of a soil sample can be determined from the familiar one-dimensional confined compression test.



$$D = \frac{\sigma_z}{\varepsilon_z}$$

Figure 6. Uniaxial Compression Test

This is shown in Figure 6. This constrained modulus may be converted to the consolidation elastic modulus E_c by the formula

$$E_c = \frac{(1+\nu)\ (1-2\nu)}{(1-\nu)} D$$

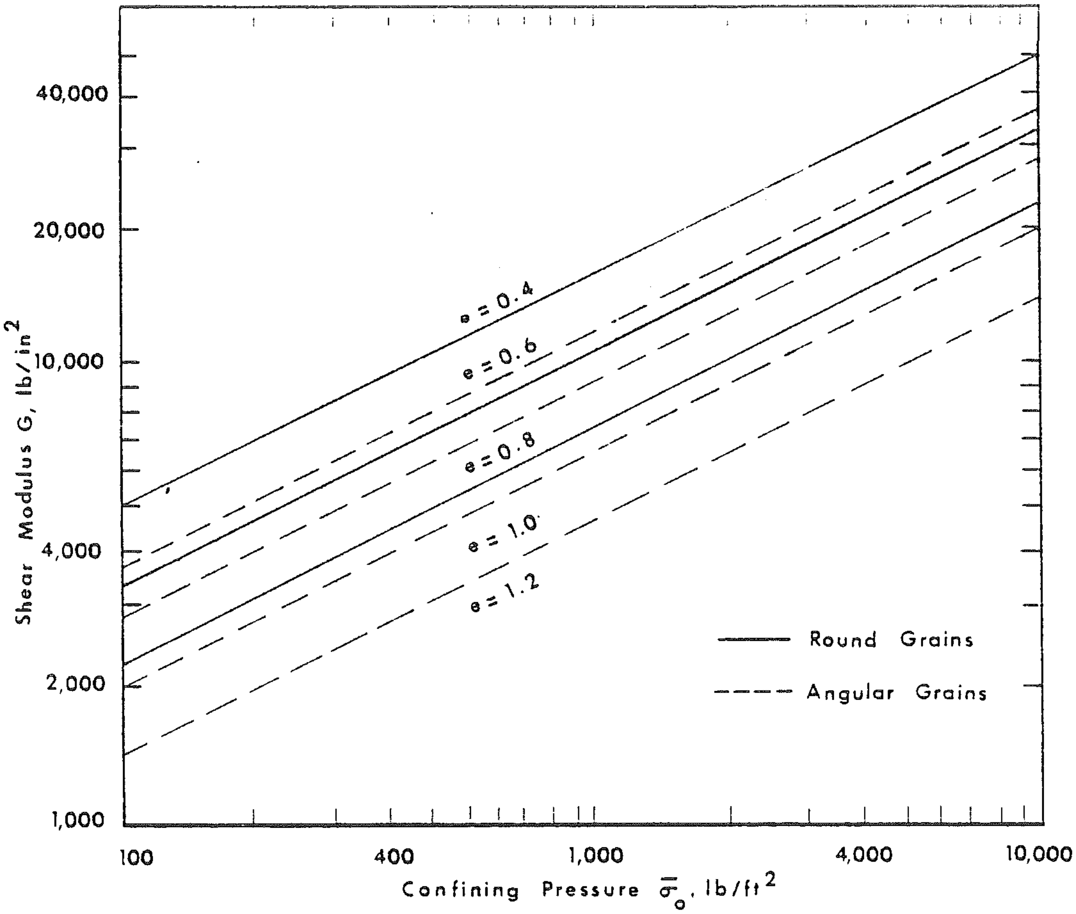


Figure 4. Variation of Shear Modulus of Dry Sands with Void Ratio and Confining Pressure (after Reference 54)

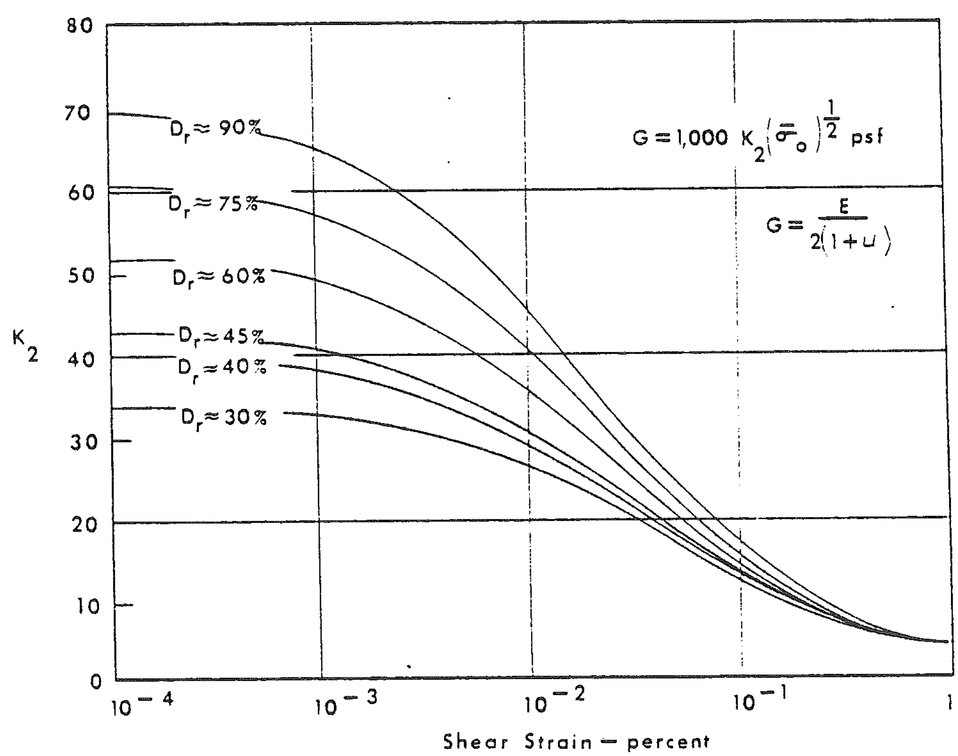


Figure 5. Shear Moduli of Sands at Different Relative Densities (after Reference 46)

An immediate disadvantage can be seen with this method. A proper conversion from the one-dimensional modulus to the three-dimensional modulus requires a very precise value of Poisson's ratio. Values of Poisson's ratio, accurate beyond the first decimal place, are very expensive to obtain, and then may be applicable only to a limited range of strains.

Balakrishna Rao (55), performed a laboratory investigation of the response of partially saturated soils in one-dimensional compression. Five typical soil types were examined, Ottawa sand, concrete sand, lean clay, fat (highly plastic) clay, and sandy clay. It was interesting to note that because of the boundary conditions imposed by the test, increasing degrees of saturation resulted in higher modulus values, probably because of limited drainage. In a field situation, the opposite would nearly always be true. Increased moisture results in increased deformation for a given load.

The uniaxial consolidation test is not recommended for determination of modulus values for pavement design for reasons stated above; namely the problems involving Poisson's ratio, the fact that the stress field and drainage conditions imposed in the test are unlike those in-place, and for many soils, because of the difficulty in obtaining a representative undisturbed sample.

1.6.4 Unconfined Compression Test

For firm, cohesive soils or cores of intact rock, the stress-strain relationship and E modulus

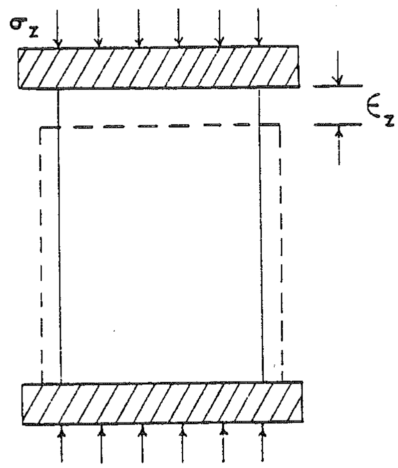


Figure 7. Unconfined Compression Test

can easily be determined from the unconfined compression test. With precise measurements of lateral strain, Poïsson's ratio can also be determined. Obviously, a disadvantage of this method is that it can only be performed on cohesive materials.

Unconfined compression tests were part of the previously cited studies by Shields (52) and Forsyth (50). Both reported low modulus values (compared to other methods) which may seem strange inasmuch as stiff samples are involved. The low values reported may be the result of the strain level or of the stress path. They may also be the result of the absence of confining pressure.

The absence of confining pressure in the unconfined compression test, like the total confinement imposed by the consolidometer, creates a situation which is not in accord with in-situ conditions. It is generally assumed that modulus increases with confining pressure, though more rapidly for sands than clays, see Figure 8.

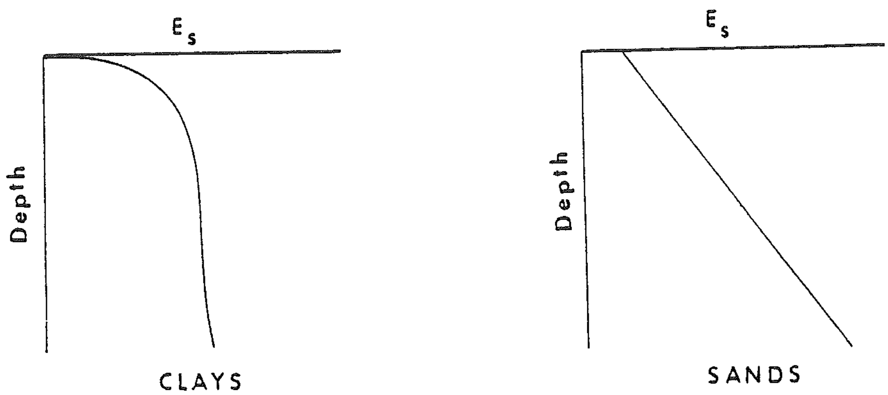


Figure 8. Effect of Confining Pressure on Subgrade Modulus

For this reason, triaxial testing is the preferred "sophisticated" means of determining modulus in the laboratory.

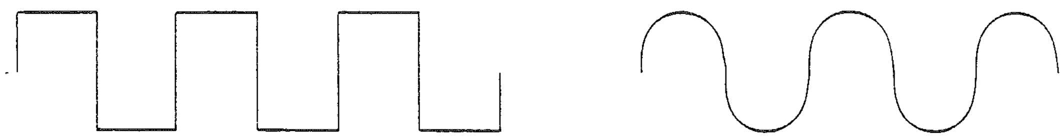
1.6.5 Triaxial Testing

A value of elastic modulus can be obtained from the conventional triaxial test in the form

$$E = \frac{\sigma_1 - \sigma_3}{\varepsilon}$$

This is not, however, the usual means of presenting triaxial data for use in pavement design technology.

The triaxial cell has been adapted for cyclic (dynamic) testing of pavement materials under conditions which simulate repetitive traffic loading. The loading program may be in the form of discrete pulses (square waveform) or a sinusoidal load of selected frequency and amplitude.



9a. Discrete Pulse   9b. Sinusoidal

Figure 9. Types of Loading

The procedure was apparently first established by Hveem (56) as a way of describing the nonlinear load response of pavement materials.

As discussed in section 1.5.6, modulus is given as M_R, the resilient modulus, which is the ratio of the repeated deviator stress to recoverable strain (see Figure 2).

It should be noted that the resilient modulus has been pursued with regard to both bituminous bound materials as well as unbound materials. In fact, the volume of test data for bound materials far outweighs that for soils. In many pavement design methodologies, M_R is considered an appropriate substitute for E_s, for all layers in the prism.

In the past ten years, a voluminous amount of laboratory data has been reported involving dynamic triaxial testing. The California DOT/University of California, Berkeley, and the U. S. Army Corps of Engineers Waterways Experiment Station have been most actively involved. Generally, several tests are performed on a given sample of material so that a regression analysis can be made.

For a given number of cycles of loading, resilient modulus is expressed in either of two ways

$$M_R = K_1 \sigma_3^{K_2}$$

or;

$$M_R = K_1 \Theta^{K_2}$$

σ_3 is the confining pressure, and Θ = sum of the three principal stresses = $\sigma_1 + 2\sigma_3$. The use of Θ is given as a function of the total stress regime.

Many investigators have published values for K_1 and K_2 for various materials. A recent compilation is available in a report by Chou (57) for granular materials.

Chisolm and Townsend (58, 59) reported that $M_R = 2570\ \Theta^{.865}$ for gravelly sand where Θ is less than 100 psi. For clays, M_R is always 5 to 12 times the static unconfined compression modulus. This is because resilient modulus measures only recoverable strain, while the modulus calculated from an unconfined compression test includes both elastic and plastic deformation.

Resilient modulus may be plotted as a function of deviator stress or number of cycles of loading

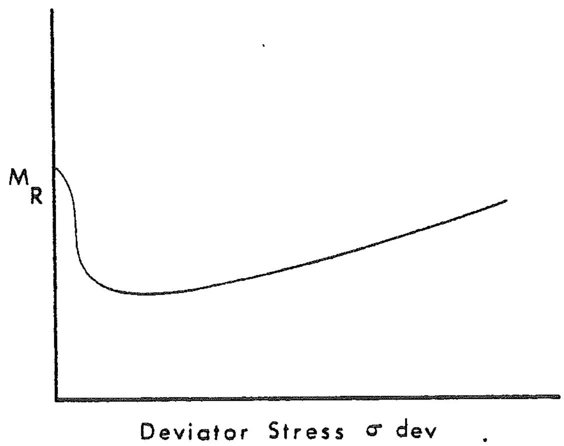


Figure 10a. Typical Relationship of Resilient Modulus as a Function of Deviator Stress

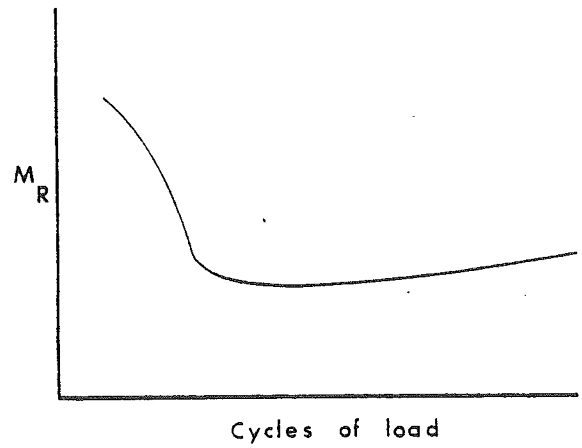


Figure 10b. Typical Relationship of Resilient Modulus as a Function of Cycles of Load

For sands, the plots appear similar. The question arises as to the deviator stress level and number of cycles of loading which are appropriate for typical pavement design problems. Kalcheff and Hicks (60), report that for sands, 50 to 100 cycles is sufficient to characterize the materials response. Probably, a deviator stress in the range of 20 to 30 psi is realistic for pavement design.

In summary, dynamic triaxial testing is the most sophisticated means of characterizing subgrade response in the laboratory today. It will undoubtedly be the subject of more research in the future. The advantages and disadvantages of using resilient modulus can be summarized as follows:

Advantages

- M_R is defined as a function of stress ($M_R = K_1\Theta^{K_2}$) and therefore carries no pretense of being an independent variable, as does E_s. The simple function shown above is almost as easy to use in a computer program as a fixed value.

- The use of M_R and repetitive testing incorporates the dynamic nature of the problem. The stress path is more like that which occurs in the prototype structure.

Disadvantages

- Repetitive triaxial testing is expensive. In order to define K_1 and K_2, it is necessary to prepare and test several specimens. This is probably not practical on a production basis.

- In any procedure involving the testing of several samples, there is likely to be random scatter in the test results. Unless the body of data is large, a realistic median value cannot be determined.

- There are boundary limitations in the triaxial testing apparatus which may introduce discrepancies which render the model a poor representation of the field situation. An example might be the effect of size of the loaded area. Another is the strain softening effect displayed by many soils under plate loading. In reality, modulus decreases as failure approaches, but the equation $M_R = K_1\theta^{K_2}$ indicates that modulus increases with increasing stress unless K_2 is negative.

Within the elastic stress range, the above equation is useful only for appropriate confining pressures.

1.6.6 R Value

R value, also called stabilometer value, is an index developed by the California Department of Transportation and used in many of the states which employ the AASHTO pavement design system.

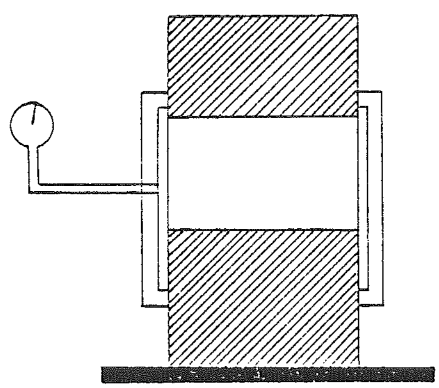


Figure 11. Stabilometer Device

R is basically a measure of resistance to deformation of a sample of material under an applied vertical pressure of 160 psi. As is the case with CBR, most laboratories serving pavement design agencies are equipped to run R values on a volume basis. Obviously, it would be desirable to have a correlation between R value and E_s.

Figure 12 shows the correlation used by the Arizona Department of Transportation. R is correlated with k, E, and Soil Support Value (SSV) which is a function of index parameters. As stated in section 1.6.2, field verification for such correlations is generally lacking. Because of problems involving boundary conditions, sample disturbance and stress path, correlations between laboratory indices and in-place response of soils are very subjective at best.

Apparently few investigators have scrutinized R value closely. Nielson, et al (61), reported correlations between k and R for clayey soils. Using a least squares curve fitting technique, they reported that $k \text{ (pci)} = 0.401 + 2.546\ R - 0.042R^2 + 0.0008R^3$.

1.6.7 CBR

The California Bearing Ratio (CBR) was adopted by the California DOT before World War II and subsequently abandoned in favor of R value. CBR is used today by a number of agencies however, including the Corps of Engineers.

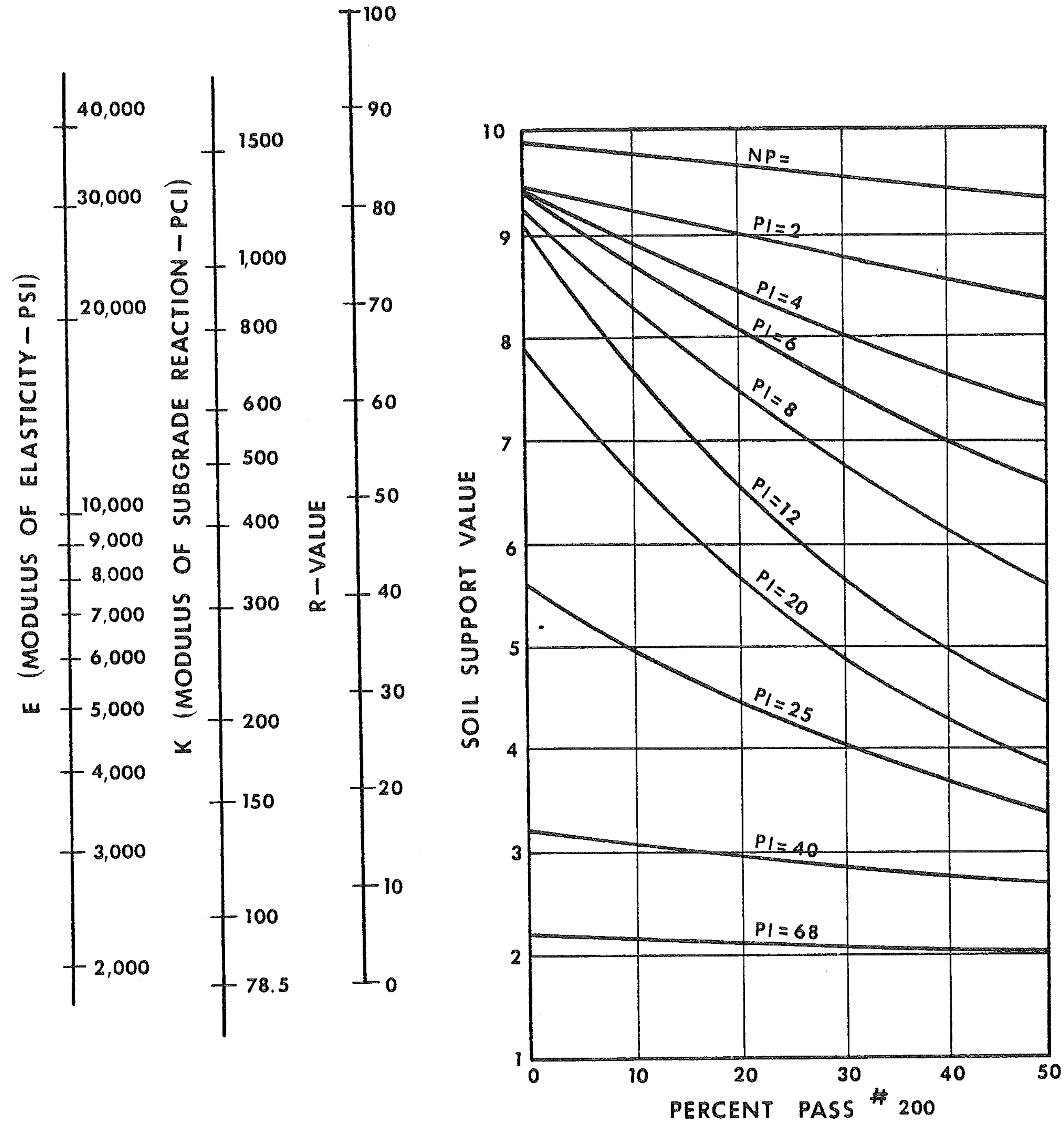


Figure 12. Soil Strength Correlation Chart used by Arizona Department of Transportation

The CBR is a penetration test. Basically, it is the ratio of the load required to cause penetration of a 3 square inch circular plunger into the soil at a specified rate compared to the load required to cause penetration at the same rate of a reference material. The test is performed both in the laboratory (in a compaction mold) and in-place in the field. Theoretically, the results should be the same, but, in reality, the confining effect of the mold may affect values for some very coarse grained materials. The samples are usually tested at a moisture condition simulating the worst possible design consideration, namely, saturation of the subgrade.

Throughout the literature concerned with pavement design, one sees the correlation attributed to Heukelom and Foster (62), that E_s = 1500 x CBR (in psi) or E_s = 10 x CBR (MN/m^2). This relationship has rarely been challenged. In actuality, Heukelom and Foster reported that E ranged from 700 to 2800 times the CBR value. For AASHTO road test subgrade materials (an A-6 soil), Marek and Dempsey (63) reported that E = 2640 x CBR in the unfrozen state. Nielson, et al (61) investigated in depth the problems of correlating E_s with CBR and R value. They found that for granular soils in the Las Cruces, New Mexico area, at 0.1 inch penetration

$$E' = 312 \text{ CBR (psi)}$$

E' is a modulus of soil reaction used in culvert analysis and is equal to 1.5 x D, the constrained modulus. This would correspond to the following set of correlations.

For ν = 0.1	E_s = 203 x CBR
0.2	E_s = 187 x CBR
0.3	E_s = 154 x CBR
0.4	E_s = 97 x CBR

Obviously there is some discrepancy among what the various investigators have found.

Kirwan and Snaith (64) state that, with regard to Irish glacial tills, considerable error resulted from trying to use a fixed ratio between CBR and resilient modulus. Likewise, Thompson and Robnett (53) reported that fine grained soils of similar CBR may exhibit widely varying values of resilient modulus.

Pichumani (65), in testing several airfield pavement subgrades, noted that the relationship E = 1500 x CBR generally resulted in much higher values of E than were derived from plate bearing tests. The major problem with CBR may be that the piston area in contact with the subgrade is so small (3 square inches) that perimeter/area ratio effects are dominant and, in many cases, mass behavior of the subgrade is not being analyzed.

1.6.8 Resonant Column

As will be discussed in detail in section 1.7, seismic velocities provide a means of calculating low strain elastic modulus values. Normally, field techniques are employed to determine shear wave velocity. (From this, G and E can be calculated if Poisson's ratio is known or assumed.) The desire for sophisticated laboratory techniques has led to the development of the resonant column.

Anderson (66) performed extensive laboratory work involving the shear velocity of cohesive soils. He noted that strain amplitude and number of cycles of loading are fundamental independent variables upon which V_s depends. Anderson also found that laboratory values of V_s were always lower than the field (in-situ) values for the same material. This was confirmed by Stoke and Richart (67) who noted that resonant column values of V_s significantly underestimated the field values. It was also noted that V_s increased with vertical confining pressure. Similar results were reported by Cunny and Fry (68). Comparative values of in-situ and laboratory determined shear and compression wave velocities from 14 widely placed sites in the continental United States showed that laboratory moduli ranged within $\pm$ 50 percent of the in-situ moduli. Resendiz, et al (69) published work dealing with E modulus of saturated clay in Mexico City. Their conclusion was that only upper and lower bounds of moduli can be determined from laboratory tests.

In summary, it must be said that the use of any of the laboratory methods described in this chapter for the determination of modulus involves some compromise and will probably require some adjustments if meaningful values of modulus are to be used for pavement design. The biggest drawbacks to laboratory testing are:

- size of specimen - invariably one uses a smaller specimen than desirable.
- difficulties in modeling correctly the boundary and environmental (i.e., soil-moisture interaction) conditions which are critical to the response of subgrades in the field.

1.7 Determination of Modulus In-Situ

1.7.1 General

It should be apparent by now that load response parameters (i.e., moduli) of earth materials are very sensitive to a number of factors. A given native deposit of soil or weathered rock could reasonably be assigned elastic modulus values spanning an entire order of magnitude, depending on the geometry, size and path of loading and on possible changes in the soil-moisture regime. The engineer seeking a k value or an E value for use in a specific analysis should give full consideration to the practical significance of these factors.

A modulus value should be calculated or chosen based on a testing procedure which closely simulates the geometry and level of deflection of the design product during its operating life. For instance, if a value of E is needed for the analysis of a vibratory machine foundation, a modulus determined dynamically at low strain level should be used. On the other hand, for pavement design, a modulus compatible with the shape and magnitude of the design wheel load is appropriate.

Experience with local conditions is essential to any engineer dealing with pavements. In almost every project where load tests were performed, or postconstruction settlement records are available, it is possible to back calculate modulus values. Likewise, a pre-design field testing program to characterize subgrade response on major projects should be encouraged. The money involved in testing will likely be overshadowed by the savings in construction costs.

1.7.2 Settlement Records - Shallow Foundations

Values obtained from programs set up to monitor settlement of structures with shallow spread-type footings of any common shape (rectangular, circular or continuous) may be inserted into appropriate equations for the deflection of loaded areas on or below the surface of an infinite elastic media. Compilation of such equations may be found in numerous references, such as in chapters 3, 4 and 5 of "Elastic Solutions for Soil and Rock Mechanics" by Poulos and Davis (70), chapter 6 of "Foundation Engineering" by Leonards (71), chapter 14 of "Soil Mechanics" by Lambe and Whitman (72), chapter 5 of "Foundation Analysis and Design" by Bowles (73), and recent papers by Mitchell and Gardner (74) and Poulos (75). In all cases, a value for Poisson's ratio of the soil must be known or assumed.

At first glance, it may seem that the preceding references contain a plethora of expressions and associated graphs and tables which have been taken from elastic theory and adapted for soil mechanics. In fact, most are of the general form $E = \frac{(1-\nu^2)p}{\rho} I$ where ρ is the settlement, p is the applied pressure and I is an "influence" factor which takes into account such variables as size and shape of the loaded area and the relative rigidity or flexibility of the loaded plate or footing.

1.7.3 Settlement Records - Pile Foundations

Modulus can be back calculated from the results of pile load tests. For contact piles built to develop both friction and end-bearing, elastic techniques developed by Poulos and Davis (75, 76) can be used.

Uncertainty arises in the interpretation of the resulting modulus. Since a thick stratum of soil is involved, the question becomes one of where the computed modulus applies. The analysis was developed for a medium whose elastic modulus is constant with depth. Thus, for a layered soil, or a soil whose modulus increases as the vertical confining pressure, such an analysis may not be reasonable. On the other hand, it may be within reason to draw a pressure bulb around the pile and use stress analysis to locate a virtual point (a point where the increase in stress is about .5 times the maximum pressure increase) and designate that the computed value of modulus be assigned there.

Nevertheless, pile load tests, while valuable for the information they give, are probably the most indecisive means of determining the subgrade modulus.

1.7.4 Plate Bearing Tests

Probably the most practical means of obtaining modulus values in-situ as input parameters in foundation or pavement design systems is the plate bearing test.

Where a plate bearing test is performed with circular plates directly on an "elastic subgrade", the general Boussinesq equation is applicable

$$E = p\,\frac{(1-\nu^2)\ 2aI}{\rho}$$

where a is the plate radius. I, the influence factor, is generally given to be 1.0 for the center of a flexible plate (uniform load) and $\pi/4 = 0.79$ for a completely rigid circular plate. Bowles (73) publishes a solution where I (rigid) is given as high as 0.88. In this report, due to the loading geometry used in the performance of plate bearing tests, modulus will be calculated using $I = 0.80$ for plates smaller than 18 inches and $I = 0.85$ for plates 24 and 30 inches in diameter where some relaxation of rigidity was likely. Compared to the changes in moduli with stress level, the differences due to influence factors are moot.

The only substantive difference between moduli determined from plate bearing tests and those from settlement data lies in the time element. Readings from plate bearing tests usually provide only primary settlement response, while settlement values from structures taken over a long time frame will incorporate secondary compression and creep potential of the soil.

Where a plate bearing test is imposed on a layered system, such as a pavement, modulus of the subgrade may also be back calculated, although a multilayered elastic analysis must be used. Several suitable methods for hand calculation are available in chapter 6 of Poulos and Davis (70). The drawback to multilayer analysis is that to calculate a modulus for the third layer, compression modulus for the upper two layers must be assumed. A sensitivity study using one of the typical three layer equations, however, showed that the final result is not sensitive to the value chosen for E_1, and only moderately sensitive to E_2 (see Figure 17).

The writer has analyzed, for the purpose of determining modulus, plate bearing tests performed by this firm at four existing airfields in the Southwest. These are given in Figures 13 through 16 and A-1-1 through A-1-8. In these, the general